## MATH 810: PROJECT II
## DUE: OCT. 9TH, 2014

(1) Exercise 5.1/page 44
(2) Exercise 5.2/page 44
**Hint:** Cover $(0,1)$ with a countable family of intervals, so that on each one of them $f = g_j$.
(3) Let $(X, \mathcal{A})$ be a measure space. Let $f : X \to [-\infty, \infty]$ (that is $f$ may be unbounded for some $x \in X$). Prove that $f$ is measurable if and only if $f^{-1}(\{-\infty\}) \in \mathcal{A}, f^{-1}(\{\infty\}) \in \mathcal{A}$ and $f : Y \to \mathbf{R}^1$ is measurable, where $Y = f^{-1}(-\infty, \infty)$.
(4) Exercise 6.4/page 50. You may use the linearity of the integral
**Hint:** Let $\epsilon > 0$. Consider a simple function $s$, so that $0 \leq s(x) \leq |f(x)|$

$$\int |f(x)| d\mu < \int s(x) d\mu + \frac{\epsilon}{2}.$$

(5) Let $(X, \mathcal{A}, \mu)$ is measure space $f \geq 0$ and measurable. Show that

$$\lambda(A) := \int_A f d\mu,$$

defined for $A \in \mathcal{A}$, is a measure. Also, show that for each $g \geq 0$, measurable, we have

$$\int g d\lambda = \int f g d\mu.$$

**Hint:** For the measure part, use Proposition 7.5. For the integral fromula, prove it first for simple functions. Extending to arbitrary positive functions, it is easy to show $\int g d\lambda \leq \int f g d\mu$. For the reverse inequality, use the approximation result Proposition 5.14 and the monotone convergence theorem.