MATH 810: PROJECT I
DUE: SEPT. 21, 2017

(1) Exercise 2.3/page 11.
Hint: I already did this in class, but fill in the details. Take $C_n = \{(p,q) : 0 < p < q < 1 : p, q = \frac{r}{s}, (r,s) = 1, r < s < n\}$ and $A_n = \sigma(C_n)$. Show that A_n are finite σ algebras, but $A = \cup_n A_n$ (which is a countable set!) is not a σ algebra.
(2) Exercise 2.6/page 11
(3) Exercise 3.5/page 16
(4) Le μ^* be an outer measure on X and A_j be a sequence of disjoint, μ^* measurable sets. Prove
$$\mu^*(E \cap \cup_n A_n) = \sum_n \mu^*(E \cap A_n).$$
Hint: One direction is easy. For the other, prove that it suffices to show
$$\mu^*(E \cap (A \cup B)) = \mu^*(E \cap A) + \mu^*(E \cap B)$$
For A, B measurable and $A \cap B = \emptyset$
(5) Exercise 4.4/page 34
(6) Exercise 4.7/page 34
(7) Exercise 4.11/page 35
Hint: Argue by contradiction. This will give a sequence $x_n \to 0$, so that $(x_n + A) \cap A = \emptyset$.
(8) Exercise 4.14/page 35
(9) Exercise 4.17/page 36
Hint: Take a countable dense set in $A : \{x_n\}_n, x_n \in A$. Show that
$$B = \cup_n [x_n - 1, x_n + 1] \cup A - 1 \cup A + 1.$$