MATH 810: PROJECT I
DUE: SEPT. 16, 2014

(1) Exercise 2.7, page 11.
(2) Exercise 3.8, page 16:
Hint: Follow the steps outlined below.
   (a) Show that $\sigma(\mathcal{A} \cup \mathcal{N}) = \{A \cup N : A \in \mathcal{A}, N \in \mathcal{N}\}$. This is nontrivial!
   (b) Define $\tilde{\mu} : \mathcal{B} \to [0, \infty]$,
$$\tilde{\mu}(A \cup N) := \mu(A).$$
   (c) Prove that $\tilde{\mu}$ is a measure (i.e. σ additivity).
   (d) Prove that all null sets for $\tilde{\mu}$ are in $\sigma(\mathcal{A} \cup \mathcal{N})$, i.e. $(X, \sigma(\mathcal{A} \cup \mathcal{N}), \tilde{\mu})$ is a complete measure space.
(3) Exercise 4.3/page 34
(4) Exercise 4.10/page 35
Hint: For every $\delta > 0$, there is a family of intervals $\{I_j\}_j$, so that $A \subset \cup I_j$ and
$$m(A) + \delta > \sum_j m(I_j)$$
On the other hand,
$$m(A) \leq \sum_j m(A \cap I_j)$$