DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 220 - Spring 2006 - MOCK EXAM 1

Your Name: ________________________________

On this exam, you may use a calculator, but no books or notes.
It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

(1) **(15 points)** Solve

$$\left|\begin{array}{l} t^3 y' + 4t^2 y = e^{-t} \\ y(-1) = 0 \end{array}\right.$$

(2) **(15 points)** Solve the initial value problem

$$\left|\begin{array}{l} x + ye^{-x}y' = 0 \\ y(0) = 1 \end{array}\right.$$

(3) **(20 points)** Solve the differential equation

$$(xe^{xy}\cos(2x) - 3)y' + (ye^{xy}\cos(2x) - 2e^{xy}\sin(2x) + 2x) = 0.$$

(4) **(15 points)** Solve

$$\left|\begin{array}{l} y'(x) = e^{x^2} - \dfrac{y}{x} \\ y(1) = -1. \end{array}\right.$$

(5) **(15 points)** Solve the initial value problem

$$\left|\begin{array}{l} y' = \frac{y^2+y}{x+1} \\ y(0) = -1/2 \end{array}\right.$$

(6) **(15 points)** Determine whether the given equation is exact. If it is exact, solve it.

(i) $(x + e^y) + (x^4 + y + 1)y' = 0$;

(ii) $(2x + y\cos x) + (\sin x - \sin y)y' = 0$.

(7) **(15 points)** Solve the initial value problem.

$$\left| \begin{array}{l} ty' + 2y = \frac{\cos t}{t} \\ y(\pi) = 0 \end{array} \right.$$

What is the domain of the solution (i.e. where is it defined?)?

(8) **(30 points)** Determine whether the given equation is exact. If it is exact, solve it.

(i) $(x + e^y) + (x^4 + y + 1)y' = 0$;

(ii) $(2x + y\cos x) + (\sin x - \sin y)y' = 0$.

Solve the initial value problem with initial condition $y(\pi) = 0$.

(9) **(35 points)** A tank contains 10 gal of water polluted with 10 lb of chemical. Water with pollutant concentration of 0.5 lb/gal starts to come in with a rate of 2 gal per minute and leaves at the same rate. How much pollutant is there at time t? What happens after a very long time?

(10) **(45 points)** Certain kind of fish population is increasing a rate proportional to the size of the population and in the absence of other factors the population doubles every 10 weeks. Assume that there were 2000 fishes in a small lake and fisherman catch 20 fishes per day, how long will it take for the population to become extict?

(11) **Extra Credit Problem (50 points)** Water is pumped into an empty tank at the constant rate of 8 gal/min and is pumped out at a rate proportional to the square of the amount of water $y(t)$ in the tank at time t (assume that the proportionality constant $k = 2$). Determine a differential equation governing the amount $y(t)$ and find out how much water is in the tank after 10 minutes. **Hint:** Consider solving the differential equation, by writing it as a differential equation for the function $v(t) = 1/y(t)$.