

**DEPARTMENT OF MATHEMATICS  
UNIVERSITY OF KANSAS  
MATH 220 - Spring 2006 - EXAM 1**

**Your Name:** \_\_\_\_\_

On this exam, you may use a calculator, but no books or notes.  
It is not sufficient to just write down the answers. You must explain how you arrived  
at your answers and how you know they are correct.

1	(20)	_____
2	(20)	_____
3	(30)	_____
4	(30)	_____
5	(20)	_____
6	(30)	_____
7	(30)	_____
Total	(150)	_____

(1) **(20 points)** Find the general solution of the differential equation

$$ty' - y = t^2e^{-t}$$

(2) ( **20 points**) Solve the initial value problem

$$\begin{cases} y' = 2y^2 + xy^2 \\ y(0) = 1, \end{cases}$$

and determine where the solution attains its minimum value.

(3) (30 points) Solve the differential equation

$$y' = \frac{2y \sin(x) - \sin(y)e^x}{\cos(y)e^x + 2 \cos(x)}.$$

- (4) **(30 points)** A tank with capacity of 600 gal originally contains 200 gal of water with 100 lbs of salt in it. Fresh water enters the tank at the rate of 4 *gal/min* and the well-stirred solution is leaving at the rate of 2 *gal/min*. Find the amount  $Q(t)$  of salt present in the tank, when the water starts to overflow (i.e. after 200 minutes).

(5) ( **20 points**) Find the solution to the initial value problem

$$\begin{cases} y'' + 4y' + 5y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

Find  $\lim_{t \rightarrow \infty} y(t)$ .

(6) **(30 points)** Solve the initial value problem

$$\begin{cases} y'' - y' - 2y = 0 \\ y(0) = \alpha \\ y'(0) = 2 \end{cases}$$

Find  $\alpha$  so that the solution approaches zero as  $t \rightarrow \infty$ .

- (7) (**Bonus problem 30 points**) NO PARTIAL CREDIT ON THE BONUS PROBLEM, I.E. ONLY FULL CREDIT OR NO CREDIT.

College graduate borrows \$8000 to buy a car. The lender charges interest of 10% per year (compounded continuously). The borrower makes payments at the constant annual rate of  $\$k$ . Determine  $k$ , so that the loan is paid in exactly three years. How much interest has the borrower paid during that three year period?