MATH 647
MOCK MIDTERM

Problem 1
Solve the $2D$ wave equation

$$\begin{array}{|c} u_{tt} = u_{xx} + u_{yy} \quad 0 \le x \le a \quad 0 \le y \le b \\ u_x(t,0,y) = 0; u_x(t,a,y) = 0; \\ u(t,x,0) = 0, u(t,x,b) = 0 \\ u(0,x,y) = f(x,y) \\ u_t(0,x,y) = 0 \end{array}$$

Problem 2
Use D'Alembert's formula to solve the one dimensional wave equation with $c = 1$ boundary value problem for a string of unit length with fixed ends, subject to $u(x,0) = \sin \pi x$, $\frac{\partial u}{\partial t}(x,0) = -5$.

Problem 3
Use the eigenfunction expansion method to solve the Poisson equation

$$\Delta u = 3u - 1$$

inside the unit square $0 < x < 1, 0 < y < 1$ with boundary conditions

$$u(0,y) = u(1,y) = 0, u(x,0) = 1, \frac{\partial u}{\partial y}(x,1) = 1.$$

Problem 4
Solve the following Laplace's equation with Robin boundary condition

$$\begin{array}{|c} u_{xx} + u_{yy} = 0 \quad 0 \le x \le a \quad 0 \le y \le b \\ u_x(t,0,y) = 0; u_x(t,a,y) = 0; \\ u_y(t,x,0) + u(t,x,0) = 0, \\ u_y(t,x,b) = 0 \end{array}$$