MATH 961: PROJECT I
DUE: SEPT. 28, 2020

(1) Problem 5.8.1/page 288.

(2) Related to the previous problem , show that if $A = A^*$ and E is an A invariant subspace of H, then we can the split $H = E \oplus E^\perp$ and define $A_E := P_E A : E \to E$ and $A_{E^\perp} = P_{E^\perp} A : E^\perp \to E^\perp$. Show that A_E and $A_{E^\perp}$ are self-adjoint as well and

$$\sigma(A) = \sigma(A_E) \cup \sigma(A_{E^\perp}).$$

(3) Let A be densely defined operator. Assume that it is closeable. Show that it has a smallest close extension, i.e. an operator $\bar{A}$ so that every close extension of A is an extension of $\bar{A}$ as well. Show that $\Gamma(\bar{A}) = \overline{\Gamma(A)}$.
Hint: For the existence of $\bar{A}$, use Zorn's lemma, with appropriate order. For every closed extension, S, the direction $\overline{\Gamma(A)} \subset \Gamma(S)$ is obvious. For the other one, construct a close extension R, with $\Gamma(R) = \overline{\Gamma(A)}$. Then, it should be $R = \bar{A}$, why?

(4) For the Hilbert space $L^2(\mathbf{R})$, define the operator $Af := xf(x)$, with domain $D(A) = \{f \in L^2(\mathbf{R}) : xf(x) \in L^2(\mathbf{R})\}$. Show that A is densely defined and closed operator. Show that $\sigma(A) = C\sigma(A) = \mathbf{R}$. Write down the resolvent $(\lambda - A)^{-1}$ for $\lambda \notin \mathbf{R}$.

(5) Problem 6.5.1 a), d), e), page 342.

(6) Problem 6.5.2/page 342.