DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MIDTERM MATH 800 - Spring 2012

Your Name: ________________________________

1	(50)	________
2	(50)	________
3	(50)	________
4	(50)	________
Total	(200)	________

(1) (50 points)
Let $f : D(0,1) \to \mathcal{C}$ be a holomorphic function. Prove that for each $r \in (0,1)$, we have

$$2f'(0) = \frac{1}{2\pi i} \int_{|z|=r} \frac{f(z) - f(-z)}{z^2} dz.$$

(2) (50 points)
Let U be the strip $\{z : -1 < \Im z < 1\}$. Assuming that $f : U \to \mathcal{C}$ is a bounded holomorphic function, satisfying

$$|f(z)| \leq A,$$

prove that for all integers n, and for all real x,

$$|f^{(n)}(x)| \leq An!$$

Give an example that shows that these estimates are sharp.

(3) (50 points)

Suppose that f is holomorphic on $D(0,1) \setminus \{0\}$.

- Let 0 be a pole for f. What is it then for f^2?
- Let 0 be an essential singularity for f. What is it for f^2?

(4) (50 points)
Let $f : D(0,1) \to \mathcal{C}$ be holomorphic. Assume that there is $0 < r_0 < 1$, so that for every $0 < r < r_0$,

$$\int_{|z|=r} f(z)\bar{z}^j dz = 0, j = 1, 2, \ldots$$

Show that

- $f(z) = 0$ on $D(0, r_0)$
- Use that $f|_{D(0,r_0)} = 0$ to conclude that $f(z) = 0$ on $D(0,1)$.
  Hint: Consider the set $E = \{z \in D(0,1) : f^{(k)}(z) = 0, k = 0, 1, \ldots\}$ and show that it is both open and close in $D(0,1)$.