

**DEPARTMENT OF MATHEMATICS
UNIVERSITY OF KANSAS
MATH 220 - SPRING 2005 - EXAM 3**

Your Name: _____

On this exam, you may use a calculator and a list of formulas.

It is not sufficient to just write down the answers. You must explain how you arrived at your answers and how you know they are correct.

1	(30)	_____
2	(20)	_____
3	(20)	_____
4	(25)	_____
5	(30)	_____
6	(25)	_____
Total	(150)	_____

- (1) **(30 points)** Find the solution of the initial value problem using the Laplace transform.

$$\begin{cases} y'' - y' - 6y = 0 \\ y(0) = 1, y'(0) = -1 \end{cases}$$

- (2) **(20 points)** For the matrix

$$A = \begin{pmatrix} \sin x & x^3 - 2x + 1 & 1 \\ x & 3x^2 - 1 & x^4 - 1 \\ \sin x + 1 & -5 \cos x & 3x - 4 \end{pmatrix}$$

compute $A'(x)$ and $\int A(x)dx$.

(3) **(20 points)** Find the inverse Laplace transform of the function

$$F(s) = \frac{se^{-3s}}{(s+1)(s^2+4)}.$$

(4) **(25 points)** Find the solution of the initial value problem using the Laplace transform.

$$\begin{cases} y'' - y' - 2y = e^t \\ y(0) = 1, y'(0) = 1 \end{cases}$$

- (5) **(30 points)** Find the solution of the initial value problem and describe its behavior for increasing t . Use Laplace transform.

$$\begin{cases} y'' + 4y = u_{\pi}(t) \\ y(0) = 0, y'(0) = 1 \end{cases}$$

- (6) **(25 points)** Find the eigenvalues and the eigenvectors of the matrix

$$\begin{vmatrix} 1 & 1 \\ 5 & -3 \end{vmatrix}$$

(7) **(30 points), BONUS PROBLEM** Solve the initial value problem using Laplace transform

$$y^{(4)} + 5y'' + 4y = 1 - u_{\pi}(t), \quad y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 0, \quad y'''(0) = 0.$$